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Analysis of elastic scattering of polarized protons by ^{6}Li using a generalized multilevel formula

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Abstract : The theory of Blatt and Biedenharn (1952) has been generalized to include the effect of adjacent multilevel. Simon's theory (1953) is also generalized to include higher degrees of polarization.

As an application the angular distribution of cross section and the vector polarization have been applied to the case of ^{6}Li$(p,p)^6$Li elastic scattering.

Keywords : Elastic scattering, polarized protons, multilevel formula

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The A = 7 mirror nuclei ^{7}Be and ^{7}Li have been subjected to many experimental and theoretical investigations [1]. Besides the general interest in developing nuclear models for the structure of these nuclei, the reaction ^{6}Li$(n,t)^4$He is the dominant breeding process for tritium in fusion reactors [2] and the capture ^{3}He$(\alpha,\gamma)^7$Be is part of the *pp* cycle and a key reaction for the understanding of the solar neutrino problem [3].

Usually, the optical model formalism is applied to scattering processes, mainly for heavy target nuclei and medium energy [4,5]. In the vicinity of resonances the optical calculations are influenced. More detailed examinations of the resonance parameters can be achieved by a complicated phase-shift analysis [6].

In this work the resonance theory, is applied to extract the energy level parameters for ^{7}Be, from the experimental cross section and polarization data of the reaction ^{6}Li (p,p).

The well known theory of Blatt and Biedenharn [7] of isolated single level formula is generalized to include the influence of N-adjacent levels. The general expression for the differential cross section is given by [8].

$$\frac{d\sigma(\alpha,\alpha')}{d\Omega} = T_{RR}(\alpha,\alpha') + T_{CC}(\alpha,\alpha') + T_{PP}(\alpha,\alpha') + T_{RC}(\alpha,\alpha')$$
$$+ T_{RP}(\alpha,\alpha') + T_{CP}(\alpha,\alpha'), \tag{1}$$

where the first three terms represent, the resonance, Coulomb and potential contributions. The other three terms represent, contributions due to interference between pairs of interactions. These terms are given by :

$$T_{RR}(\alpha,\alpha') = \frac{\lambda_\alpha^2}{(2i+1)(2I+1)} \sum_{n=1}^{N} \sum_{m=n}^{N} \sum_{l_1(n) l_2(m) l_1'(n) l_2'(m)} (-)^{s-s'l_1(n)-l_2(m)+l_1'(n)-l_2'(m)}$$
$$\frac{g_{\alpha l_1(n)s}\, g_{\alpha l_2(m)s}\, g_{\alpha l_1'(n)s'}\, g_{\alpha l_2'(m)s'}}{(1+\delta_{m,n})^2 \left[(E-E_0(n))^2 + (1/2\,\Gamma(n))^2\right]^{1/2} \left[(E-E_0(m))^2 + (1/2\,\Gamma(m))^2\right]^{1/2}}$$
$$\times Z\big(l_1(n)J(n)l_2(n)J(n), sL\big)\, Z\big(l_1'(n)J(n)l_2'(n)J(n), s'L\big)$$
$$\times \cos\Big(\psi_{l_1}(n) + \psi_{l_1'}(n) + \phi_{l_1}(n) + \phi_{l_1'}(n) + \beta(n)$$
$$- \psi_{l_2}(m) - \psi_{l_2'}(m) - \phi_{l_2}(m) - \phi_{l_2'}(m) - \beta(m)\Big) P_L \cos(\theta) \tag{2}$$

$$T_{CC}(\alpha,\alpha') = Z^2 \operatorname{cosec}^4\left(\frac{\theta}{2}\right), \tag{3}$$

$$T_{PP}(\alpha,\alpha') = \sum_{n=1}^{N} \frac{\lambda_\alpha^2 (2J(n)+1)}{(2I+1)(2i+1)} \sum_{l=0} \sum_{l'=0} \sum_{L=|l+l'|}^{l+l'} (2l+1)(2l'+1)(ll'00|ll'L0)$$
$$\sin\phi_l \sin\phi_l' \cos(2\psi_l + \phi_l - 2\psi_l' + \phi_l') P_L \cos(\theta), \tag{4}$$

$$T_{RC}(\alpha,\alpha') = \sum_{n=1}^{N} \frac{\lambda_\alpha^2 (2J(n)+1)}{(2I+1)(2i+1)} \sum_{l=0} \sum_{l'=0} \sum_{l=l_{min}}^{J(n)+I+i} \frac{g_{\alpha l_1(n)} g_{\alpha' l_1'(n)}}{\left[(E-E_0(n))^2 + (1/2\,\Gamma(n))^2\right]^{1/2}}$$
$$\operatorname{cosec}^2\left(\frac{\theta}{2}\right) \times \sin(2\zeta \ln \sin\frac{\theta}{2} + 2\psi_l + 2\phi_l + \beta(n)) P_L \cos(\theta), \tag{5}$$

$$T_{RP}(\alpha,\alpha') = -\sum_{n=1}^{N} \frac{\lambda_\alpha^2 (2J(n)+1)}{(2I+1)(2i+1)} \sum_{l=0} \sum_{l'=0} \sum_{L=|l+l'|}^{l+l'} (2l'+1)(ll'00|ll'L0)^2$$
$$\times \frac{g_{\alpha l_1(n)} g_{\alpha' l_1'(n)}}{\left[(E-E_0(n))^2 + (1/2\,\Gamma(n))^2\right]^{1/2}}$$
$$\times \sin\phi_{l'} \sin(2\psi_l + 2\phi_l - \beta(n) - 2\psi_{l'} - \phi_{l'}) P_L \cos(\theta), \tag{6}$$

$$T_{CP}(\alpha,\alpha') = -\lambda_\alpha^2 Z \operatorname{cosec}^2\left(\frac{\theta}{2}\right)\sum_{l=0}^{\infty}(2l+1)\sin\phi_l$$

$$\cos(2\zeta \ln\sin\frac{\theta}{2} + 2\psi_l + \phi_l)P_l\cos(\theta), \tag{7}$$

and with the usual notations of the functions as defined in ref. [7]. It is to be noted that the interference between adjacent resonance levels is included in the resonance term of eq. 2.

By introducing the Coulomb, potential and resonance matrices of Goldfarb and Rook [9] in Simon's formula of polarization [10], an explicit formula for polarization can be obtained [11], restricting ourselves to the elastic processes. For the case of polarized projectile and target, the analyzing power takes the form

$$\left\langle T_{\mu'}^{q'}\right\rangle = \frac{\lambda_\alpha^2}{4(2I+1)(2q'+1)^{1/2}}\sum(-)^{\mu'+I-i-s-q'}X(iai,s_1qs_2,IbI)$$

$$T_{\mu'}^{q'}T_{\mu-\lambda}^{b}(I)\,(ab\mu-\lambda\,|\,abq0)\,D_{\mu,\mu'}^{L}(\phi,\theta,0)$$

$$\times\left[\sum_{s_2'}\sum_{l_2'(n)}(-)^{s_1'-s_2}W(is_1'Is_2',Iq')\{R_{RR}(\alpha,\alpha')+R_{RP}(\alpha,\alpha')+R_{RC}(\alpha,\alpha')\}\right]$$

$$+\left[T_{CC}(\alpha,\alpha')+T_{CP}(\alpha,\alpha')+T_{PP}(\alpha,\alpha')\right]\delta_{q0}\delta_{\mu0}\delta_{q'0}\delta_{\mu'0}, \tag{8}$$

where $R_{RR'}$, R_{RP} and R_{RC} correspond to reaction matrices for resonance and interference of resonance with both potential and Coulomb effects respectively. These terms are given by :

$$R_{RR}(\alpha,\alpha') = \sum_{n=1}^{N}\sum_{m=n}^{N}(1+\delta_{n,m})i^{l_2'(m)-l_2(m)+l_1(n)-l_1'(n)}$$

$$\frac{g_{\alpha l_1(n)s_1}g_{\alpha l_1'(n)s_1'}g_{\alpha l_2(n)s_2}g_{\alpha l_2'(n)s_2'}}{\left[(E-E_0(n))^2+(1/2\,\Gamma(n))^2\right]^{1/2}\left[(E-E_0(m))^2+(1/2\,\Gamma(m))^2\right]^{1/2}}$$

$$\times G_\mu^*\big(J(n)l_1(n)sI,Lq,J(m)l_2(m)s_2\big)G_\mu'\big(J(n)l_1'(n)sI',Lq',J(m)l_2'(m)s_2'\big)$$

$$\times i_{\sin}^{\cos}\Big(\psi_{l_2}(m)+\psi_{l_2'}(m)+\phi_{l_2}(m)+\phi_{l_2'}(m)-\psi_{l_1}(n)-\psi_{l_1'}(n)$$

$$-\phi_{l_1}(n)-\phi_{l_1'}(n)+\beta(m)-\beta(n)\Big), \tag{9}$$

$$R_{RP}(\alpha, \alpha') = \pm 4\sum_{n=1}^{N} i^{l_1(n)-l'_1(n)} W(is'_1 is_2, lq') \frac{g_{\alpha l_1}(n)_{s_1} g_{\alpha' l'_1}(n)_{s'_1}}{\left[(E-E_0(n))^2 + (1/2\,\Gamma(n))^2\right]^{1/2}} \sin(\phi_{l_2})$$

$$\times G^*_{\mu}\left(J(n)l_1(n)s_1; Lq; J_2 l_2 s_2\right) G_{\mu'}\left(J(n)l'_1 s'_1; Lq; J_2 l_2 s_2\right)$$

$$\times i^{\sin}_{\cos}\left(2\psi_{l_2}(n) + \phi_{l_2}(n)\psi_{l_1}(n) - \psi_{l'_1}(n) - \phi_{l_1}(n) - \phi_{l'_1}(n) - \beta(n)\right) \quad (10)$$

$$R_{RC}(\alpha, \alpha') = \pm 4\sum_{n=1}^{N} i^{l_1(n)-l'_1(n)} W(is'_1 is_2, lq') \frac{g_{\alpha l_1}(n)_{s_1} g_{\alpha' l'_1}(n)_{s'_1}}{\left[(E-E_0(n))^2 + (1/2\,\Gamma(n))^2\right]^{1/2}} \sin(\sigma_0 + \psi_{l_2})$$

$$\times G^*_{\mu}\left(J(n)l_1(n)s_1; Lq; J_2 l_2 s_2\right) G_{\mu'}\left(J(n)l'_1 s'_1; Lq; J_2 l_2 s_2\right)$$

$$\times i^{\sin}_{\cos}\left(\psi_{l_2}(n) - \psi_{l_1}(n)\psi_{l'_1}(n) - \sigma_0 - \phi_{l'_1}(n) - \beta(n) - \phi_{l_1}(n)\right) \quad (11)$$

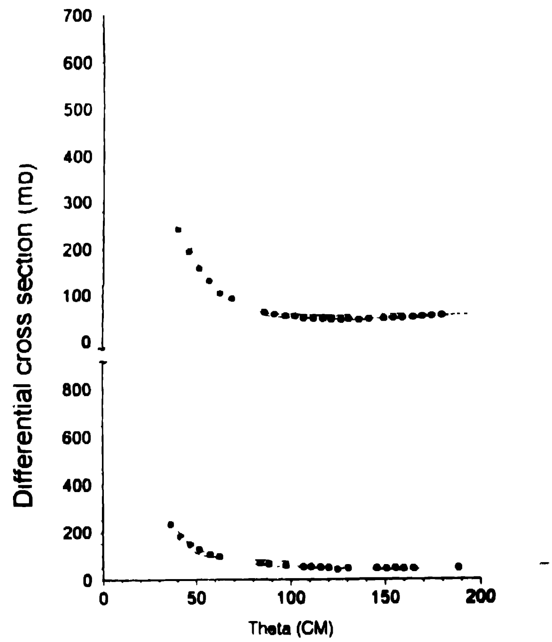

Figure 1. Angular distribution of elastic scattering of polarized protons by ^{6}Li. The upper and lower figures are for $E_p = 4.0$ and 4.6 MeV. The experimental points are due to ref. [2] The dashed lines are optical model fit [2]. The closed curves are the results of present work

The upper and lower signs and functions correspond to even and odd $q + q'$ cases respectively. Here the sum in the first term of eq. 8 is over all possible combinations of quantum numbers

$$l_1(n),\ l_2(n),\ l_1'(n),\ l_2'(n),\ s_1,\ s_2,\ a,\ b, \text{ and } L. \tag{12}$$

A computer code has been developed to compute the expressions for both of the elastic scattering cross section and polarization, where the polarization factor is given by

$$\bar{P}(\theta) = \frac{\sqrt{6}\langle T_1^1 \rangle}{\frac{\partial \sigma}{\partial \Omega}} \tag{13}$$

and the analyzing power is given by

$$A = P_t \bar{P}(\theta), \tag{14}$$

where P_t is the polarization of the incident particle.

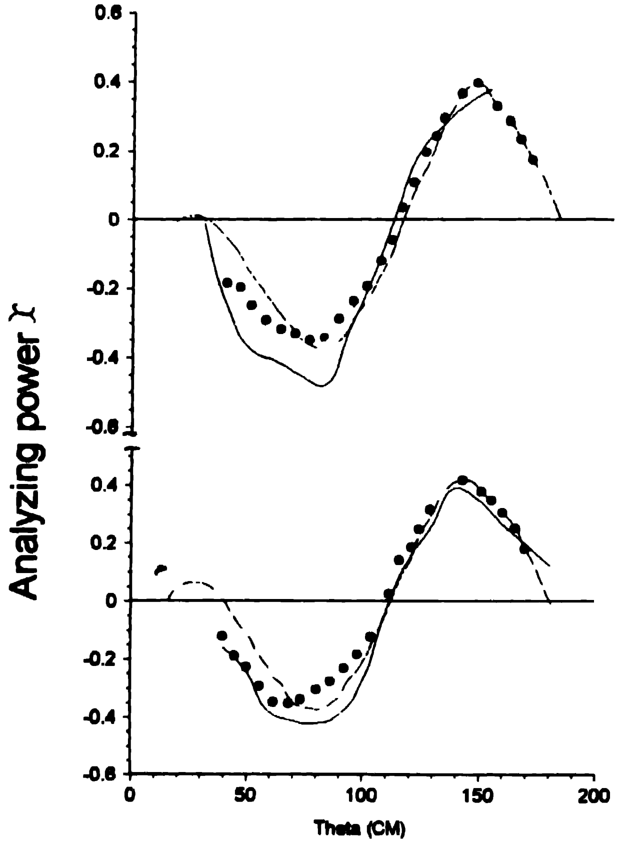

Figure 2. Analysing power of elastic scattering of polarized protons by ^{6}Li. Captions are the same as Figure 1.

Analysis of differential cross section and analyzing power have been carried out at E_p = 4 and 4.6 MeV. In this energy range the nearly resonance levels are shown in Table 1.

Table 1. Energy levels of ^{7}Be used in analysis together with the partial widths obtained from the fitting

E_x (MeV)	Γ (MeV)	Γ_p (MeV)	Γ_p/Γ	J^π
9 29	1.28	0.32	0.25	$1/2^-$
9 80	2 23	1.68	0 753	$1/2^-$
9 95	1 47	0 4	0.235	$5/2^+$
12 34	4 74	1 77	0.373	$3/2^-$

The reduced widths are used as variable parameters and are iterated to reproduce the experimental data. The results are shown in the Table 1. Figures 1, 2 show the fit obtained with the resonance theory (solid line) together with that obtained with optical model analysis (dashed line). The fit obtained is quite satisfactory taking into consideration that the search is much more restrictive than that of the optical model.

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